

Lattice perturbation theory in the overlap formulation for the Yukawa and gauge interactions

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ABSTRACT

Lattice perturbation theory is discussed in the overlap formulation for the Yukawa and gauge interactions. One and two point functions are studied for fermion, scalar and gauge fields, taking the Standard Model as an example. The formulae for the self-energies are given from which their divergent and finite parts can be computed at the one loop level.

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1 Introduction

The overlap formulation [1] and Shamir type fermion [2, 3] are new prescriptions for regularizing a chiral fermion on a lattice. Recent analytical and numerical checks on the fermion chirality in these formulations [4]-[8] suggest that vector-like theories will be regularized preserving chiral symmetry, though there remain subtle points still to be clarified on the gauge invariance (and accordingly the renormalizability) for a regularization of a general chiral gauge theory. The existence of chiral symmetry allows a clearer study of the phenomenon of chiral symmetry breaking in a vector like gauge theory and the effects of the strong Yukawa couplings in Higgs models, and provides a possibility to define a chiral gauge theory in a non-perturbative way.

In this paper we discuss lattice perturbation theory for the Yukawa and gauge interactions in the overlap formulation. To be realistic, we take the Standard Model as an example, in which Yukawa interactions, chiral and vector like gauge interactions are involved, and we analyze the one and two point functions for fermions, scalars and gauge bosons. We also give the formulae for the self-energies at the one loop level, from which both the divergent and finite parts can be computed. Lattice perturbation theory for the Yukawa interactions has not been discussed before in this formulation. For the gauge interactions, our study supplements the existing analyses [1] [8]-[12], as will be seen later.

The rest of the paper is organized as follows. In Sec. 2, we define our model on a lattice, following the notation of Refs. [8, 9], and discuss the lattice perturbation theory of the Yukawa interactions, as well as that of the gauge interactions in the overlap formulation. Both chiral and Dirac fermions are considered. In Sec. 3, we study the effects of the Yukawa couplings. The tadpole diagram and the self-energies of the Higgs boson and the fermion are studied. We confirm the relation between the tadpole term and the self-energy of the Nambu-Goldstone boson. In Sec. 4, we discuss the effects of the gauge interactions and compute the vacuum polarization and fermion self-energy. We demonstrate the cancellation of the quadratic divergence in the vacuum polarization. In these two sections, we show that the ultraviolet divergences are correctly reproduced and the fermion mass is renormalized multiplicatively. The general formulae for the self-energies of the fermions and bosons (both scalar and gauge bosons) are given in appendix

A. In appendix B, we briefly review the lattice perturbation of the $SU(2) \times U(1)$ Higgs doublets.

2 Formalism

In the overlap formulation, the effective action due to chiral fermions in the presence of gauge and scalar fields is expressed by the overlap of the two vacua $|A\pm\rangle$ of the two Hamiltonians \mathcal{H}_\pm . These two Hamiltonians are different from each other only in the sign of the mass term, and are derived from the theory describing 4+1 dimensional Dirac fermions, which simulate chiral fermions in 4 dimensions. Consider a multiplets ψ of massive Dirac fermions in 4+1 dimensions, interacting with the Higgs doublet

$$\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix} = \begin{pmatrix} i\pi_+(x) \\ \{v + H(x) - i\pi_3(x)\}/\sqrt{2} \end{pmatrix}. \quad (2.1)$$

Here the Higgs fields do not depend on the fifth coordinate (which is the time in 4+1 dimensional space-time). The Hamiltonians are given by

$$\mathcal{H}_\pm = \int d^4x \psi^\dagger(x) \gamma_5 \left[\sum_{\mu=1}^4 \gamma_\mu \partial_\mu \pm T_c \Lambda + H_Y(x) \right] \psi(x), \quad (2.2)$$

where the mass term Λ corresponds to the height of the domain wall in the original domain wall fermion [13, 14], T_c is the chirality matrix which determines the chirality of each components of ψ , and $H_Y(x)$ describes the interaction of the chiral multiplet with Higgs scalars [9]. For the lepton sector of the standard model,

$$\psi = \begin{pmatrix} \nu_L \\ e_L \\ e_R \end{pmatrix}, \quad T_c = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad H_Y = y_l \begin{pmatrix} 0 & 0 & \phi_+ \\ 0 & 0 & \phi_0 \\ \phi_+^* & \phi_0^* & 0 \end{pmatrix}, \quad (2.3)$$

and for the quark sector of the standard model,

$$\psi = \begin{pmatrix} u_L \\ d_L \\ u_R \\ d_R \end{pmatrix}, \quad T_c = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$H_Y = y_u \begin{pmatrix} 0 & 0 & \phi_0^* & 0 \\ 0 & 0 & -\phi_+^* & 0 \\ \phi_0 & -\phi_+ & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + y_d \begin{pmatrix} 0 & 0 & 0 & \phi_+ \\ 0 & 0 & 0 & \phi_0 \\ 0 & 0 & 0 & 0 \\ \phi_+^* & \phi_0^* & 0 & 0 \end{pmatrix}, \quad (2.4)$$

where y_l , y_u and y_d are the Yukawa couplings. Since the Higgs sector connects the left-handed fermions to the right-handed fermions and vice versa, the chirality matrix T_c anti-commutes with H_Y ; $\{T_c, H_Y\} = 0$. From the Hamiltonians (2.2), the gauge invariant Hamiltonians on the lattice are obtained as

$$\begin{aligned} \mathcal{H}_\pm = & a^4 \sum_n \psi_n^\dagger \gamma_5 \left[\frac{1}{2a} \sum_\mu [\gamma_\mu \{W_{n,n+\hat{\mu}} \psi_{n+\hat{\mu}} - W_{n,n-\hat{\mu}} \psi_{n-\hat{\mu}}\} \right. \\ & \left. + T_c \left\{ \lambda/a \psi_n \pm \frac{r}{2a} \sum_\mu (W_{n,n+\hat{\mu}} \psi_{n+\hat{\mu}} + W_{n,n-\hat{\mu}} \psi_{n-\hat{\mu}} - 2\psi_n) \right\} + H_Y \psi_n \right], \end{aligned} \quad (2.5)$$

by discretising Eq. (2.2), adding the Wilson term and inserting the link variable. The link variable $W_{n,n+\hat{\mu}}$ is defined as $W_{n,n+\hat{\mu}} = U_{n,n+\hat{\mu}} V_{n,n+\hat{\mu}}$, where $U_{n,n+\hat{\mu}}$ and $V_{n,n+\hat{\mu}}$ are the link variables of the $SU(2)$ and $U(1)$ gauge interactions, respectively. They commute with each other and satisfy the relations $U_{n,n+\hat{\mu}} = U_{n+\hat{\mu},n}^\dagger$ and $V_{n,n+\hat{\mu}} = V_{n+\hat{\mu},n}^\dagger$.

To quantize the system, we set the commutation relations,

$$\{\psi_{\alpha m}, \psi_{\beta n}^\dagger\} = \frac{1}{a^4} \delta_{\alpha\beta} \delta_{mn}, \quad \{\psi_{\alpha m}, \psi_{\beta n}\} = \{\psi_{\alpha m}^\dagger, \psi_{\beta n}^\dagger\} = 0, \quad (2.6)$$

which are the equal time commutation relations for Dirac fermions in 4+1 dimensions.

To develop the perturbation theory, we divide the Hamiltonian (2.5) into the free part and the interactions, and go into the momentum space. H_Y is divided into the mass matrix M and the interaction part as $H_Y = M + Y_n$, $M = \langle H_Y \rangle$, where $\langle H_Y \rangle$ contains only the vacuum expectation value of the doublet (2.1). The link variables are expanded in terms of the gauge couplings g and g' using the identifications,

$$U_{n,n+\hat{\mu}} = e^{iga \sum_i T^i W_\mu^i(n)}, \quad V_{n,n+\hat{\mu}} = e^{ig'a Y B_\mu(n)}, \quad (2.7)$$

where T^i and Y are the generators of the $SU(2)$ and $U(1)$ gauge group corresponding to the multiplet ψ . For the lepton sector, T^i and Y are three by three matrices given by

$$T^i = \begin{pmatrix} \frac{1}{2}\sigma_i & 0 \\ 0 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} Y_L & 0 & 0 \\ 0 & Y_L & 0 \\ 0 & 0 & Y_{l_R} \end{pmatrix}, \quad (2.8)$$

and for the quark sector, T^i and Y are four by four matrices given by

$$T^i = \begin{pmatrix} \frac{1}{2}\sigma_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} Y_Q & 0 & 0 & 0 \\ 0 & Y_Q & 0 & 0 \\ 0 & 0 & Y_{u_R} & 0 \\ 0 & 0 & 0 & Y_{d_R} \end{pmatrix}, \quad (2.9)$$

where σ_i are the Pauli matrices and $Y_{l_L} = -1/2$, $Y_{l_R} = -1$, $Y_Q = 1/6$, $Y_{u_R} = 2/3$ and $Y_{d_R} = -1/3$. Performing the following Fourier transformations,

$$\psi_n = \int_p \psi(p) e^{ipn}, \quad \bar{\psi}_n = \int_q \bar{\psi}(q) e^{-iqn}, \quad A_\mu^i(n) = \int_p A_\mu^i(p) e^{ipa(n+\hat{\mu}/2)}, \quad Y_n = \int_p Y(p) e^{ipn}, \quad (2.10)$$

the Hamiltonians (2.5) are divided into the free part and the interaction part as,

$$\mathcal{H}_\pm(A, \Phi) = \int_p \psi^\dagger(p) H_\pm(p) \psi(p) + \mathcal{V}(A, \Phi), \quad (2.11)$$

$$H_\pm(p) = \gamma_5 \left[\sum_\mu i \tilde{p}_\mu \gamma_\mu + T_c X_\pm(p) + M \right], \quad X_\pm(p) = \pm \frac{\lambda}{a} + \frac{ar}{2} \hat{p}^2, \quad (2.12)$$

where $\tilde{p}_\mu = (1/a) \sin(p_\mu a)$, $\hat{p}_\mu = (2/a) \sin(p_\mu a/2)$ and the momentum integral is over the Brillouin zone $[-\pi/a, \pi/a]$.

First we consider the free part and then take the interactions into account as perturbations. To quantize the free part, we expand the operator $\psi(p)$ in terms of the creation and annihilation operators as

$$\psi(p) = \sum_{f, \sigma_f} \left[u_\pm(p, f, \sigma_f) b_\pm(p, f, \sigma_f) + v_\pm(p, f, \sigma_f) d_\pm^\dagger(p, f, \sigma_f) \right], \quad (2.13)$$

where $u_\pm(p, f, \sigma_f)$ and $v_\pm(p, f, \sigma_f)$ are the eigenspinors of the one-particle free Hamiltonians $H_\pm(p)$ and satisfy the eigenequations

$$H_\pm(p) u_\pm(p, f, \sigma_f) = \omega_\pm(p, f) u_\pm(p, f, \sigma_f), \quad H_\pm(p) v_\pm(p, f, \sigma_f) = -\omega_\pm(p, f) v_\pm(p, f, \sigma_f), \\ \omega_\pm(p, f) = \sqrt{\tilde{p}^2 + m_f^2 + X_\pm^2(p)}. \quad (2.14)$$

Here m_f^2 are the eigenvalues of the M^2 and we denote the flavor indices of the spinors as f and their spin indices as σ_f . For example, $f = \nu_L, e_L$ and e_R for leptons. In Eq. (2.14), the relations $m_{e_L} = m_{e_R}$ and $\omega_\pm(p, e_L) = \omega_\pm(p, e_R)$ hold. (Later we will introduce the propagator for the Dirac fermion with the index e_V , which is the sum of the propagators of e_L and e_R .) The explicit expressions of the eigenspinors are

$$u_\pm(p, f, \sigma_f) = \frac{\{\omega_\pm(p, f) + H_\pm(p)\}}{\sqrt{2\omega_\pm(\omega_\pm + X_\pm)}} \chi(f, \sigma_f), \\ v_\pm(p, f, \sigma_f) = \frac{\{\omega_\pm(p, f) - H_\pm(p)\}}{\sqrt{2\omega_\pm(\omega_\pm - X_\pm)}} \chi(f, \sigma_f), \quad (2.15)$$

where the spinors $\chi(f, \sigma_f)$ satisfy $\gamma_5 T_c \chi(f, \sigma_f) = \chi(f, \sigma_f)$ and $M^2 \chi(f, \sigma_f) = m_f^2 \chi(f, \sigma_f)$.

The orthonormality conditions for these spinors are

$$u_{\pm}(p, f, \sigma_f)^{\dagger} u_{\pm}(p, g, \sigma_g) = v_{\pm}(p, f, \sigma_f)^{\dagger} v_{\pm}(p, g, \sigma_g) = \delta_{fg} \delta_{\sigma_f \sigma_g}, \quad u_{\pm}(p, f, \sigma_f)^{\dagger} v_{\pm}(p, g, \sigma_g) = 0. \quad (2.16)$$

The creation annihilation operators (b_+, d_+) satisfy the commutation relations

$$\begin{aligned} \{b_+(p, f, \sigma_f), b_+^{\dagger}(q, g, \sigma_g)\} &= \{d_+(p, f, \sigma_f), d_+^{\dagger}(q, g, \sigma_g)\} = (2\pi)^4 \delta_{f,g} \delta_{\sigma_f \sigma_g} \delta_P^4(p - q), \\ \{b_+(p, f, \sigma_f), d_+^{\dagger}(q, g, \sigma_g)\} &= \{b_+(p, f, \sigma_f), d_+(q, g, \sigma_g)\} = 0, \end{aligned} \quad (2.17)$$

where $\delta_P(p - q)$ is the periodic δ -function on the lattice. (The same relations hold also for (b_-, d_-)). The two spinor basis (u_+, v_+) and (u_-, v_-) are related by the transformations,

$$\begin{aligned} u_-(p, f, \sigma_f) &= \cos \beta(p, f) u_+(p, f, \sigma_f) - \sin \beta(p, f) v_+(p, f, \sigma_f), \\ v_-(p, f, \sigma_f) &= \sin \beta(p, f) u_+(p, f, \sigma_f) + \cos \beta(p, f) v_+(p, f, \sigma_f), \end{aligned} \quad (2.18)$$

where $\cos \beta(p, f) = u_+^{\dagger}(p, f, \sigma_f) u_-(p, f, \sigma_f)$ is given by,

$$\cos \beta(p, f) = \frac{1}{\sqrt{2\omega_+ 2\omega_-}} [\sqrt{(\omega_+ + X_+)(\omega_- + X_-)} + \sqrt{(\omega_+ - X_+)(\omega_- - X_-)}]. \quad (2.19)$$

Eqs. (2.18) lead to the Bogoluibov transformation

$$\begin{aligned} b_-(p, f, \sigma_f) &= \cos \beta(p, f) b_+(p, f, \sigma_f) - \sin \beta(p, f) d_+^{\dagger}(p, f, \sigma_f), \\ d_-^{\dagger}(p, f, \sigma_f) &= \sin \beta(p, f) b_+(p, f, \sigma_f) + \cos \beta(p, f) d_+^{\dagger}(p, f, \sigma_f). \end{aligned} \quad (2.20)$$

between the two basis (b_+, d_+) and (b_-, d_-) .

The free propagator is defined by the vacuum expectation value of the operator $\Omega(p, q) = \{\psi(p) \bar{\psi}(q) - \bar{\psi}(q) \psi(p)\}/2$ as,

$$\begin{aligned} \langle + | \Omega(p, q) | - \rangle &= (2\pi)^4 \delta_P^4(p - q) \sum_f \frac{1}{2} [S_+(p, f) - S_-(p, f)], \\ S_+(p, f) &= \frac{1}{\cos \beta(p, f)} \sum_{\sigma_f} u_+(p, f, \sigma_f) \bar{u}_-(p, f, \sigma_f), \\ S_-(p, f) &= \frac{1}{\cos \beta(p, f)} \sum_{\sigma_f} v_-(p, f, \sigma_f) \bar{v}_+(p, f, \sigma_f). \end{aligned} \quad (2.21)$$

The pole of the propagator is the zero of $\cos \beta(p, f)$, which occurs if both the two relations $\omega_+ = X_+$ and $\omega_- = -X_-$ are satisfied. This condition is fulfilled only at the center of the Brillouin zone. Since $\cos \beta(p, f_L) = \cos \beta(p, f_R)$, it is convenient to introduce the Dirac type propagator $S_\pm(p, f_V) = S_\pm(p, f_L) + S_\pm(p, f_R)$.

The spinors u_\pm and v_\pm have the following continuum limits at the center of the Brillouin zone:

$$\begin{aligned} u_+(p, f, \sigma_f) &= [1 + \frac{a}{2\lambda}\{i\not{p} + M\} + \mathcal{O}(a^2)]\chi(f, \sigma_f), \\ u_-(p, f, \sigma_f) &= \frac{1}{\sqrt{p^2 + m_f^2}}[\{i\not{p} + M\} + \frac{a}{2\lambda}(p^2 + m_f^2) + \mathcal{O}(a^2)]\chi(f, \sigma_f), \\ v_-(p, f, \sigma_f) &= [1 + \frac{a}{2\lambda}\{i\not{p} + M\} + \mathcal{O}(a^2)]\chi(f, \sigma_f), \\ v_+(p, f, \sigma_f) &= \frac{1}{\sqrt{p^2 + m_f^2}}[-\{i\not{p} + M\} + \frac{a}{2\lambda}(p^2 + m_f^2) + \mathcal{O}(a^2)]\chi(f, \sigma_f), \end{aligned} \quad (2.22)$$

and the continuum limit of $\cos \beta(p, f)$ is

$$\cos \beta(p, f) = \frac{a}{\lambda}\sqrt{p^2 + m_f^2} + \mathcal{O}(a^3). \quad (2.23)$$

For the lepton sector, the continuum limits of the propagators are

$$\begin{aligned} S_\pm(p, \nu_L) &= \frac{\lambda}{a} \frac{1}{p^2} \begin{pmatrix} \mp P_L i\not{p} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ S_\pm(p, e_V) &= \frac{\lambda}{a} \frac{1}{p^2 + m_e^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mp P_L i\not{p} & \pm P_L m_e \\ 0 & \pm P_R m_e & \mp P_R i\not{p} \end{pmatrix}, \end{aligned} \quad (2.24)$$

where $P_{L(R)} = (1 - (+)\gamma_5)/2$.

Next consider the interaction parts. The Yukawa interaction is given by

$$V_Y = a^4 \sum_n \bar{\psi}_n Y_n \psi_n = \int_{p,q} \bar{\psi}(p) Y(p - q) \psi(q). \quad (2.25)$$

For the lepton sector,

$$Y(p) = y_l \begin{pmatrix} 0 & 0 & i\pi_+(p) \\ 0 & 0 & \{H(p) - i\pi_3(p)\}/\sqrt{2} \\ -i\pi_+^*(p) & \{H(p) + i\pi_3(p)\}/\sqrt{2} & 0 \end{pmatrix}. \quad (2.26)$$

The gauge interactions are obtained from the expansion of the link variable

$$\begin{aligned}
W_{n,n+\hat{\mu}} &= 1 + ia \left[eQ A_\mu(n) + g_z(T^3 - s_w^2 Q) Z_\mu(n) + \frac{g}{\sqrt{2}} \{T_+ W_\mu^+(n) + T_- W_\mu^-(n)\} \right] \\
&+ \frac{1}{2!} (ia)^2 \left[eQ A_\mu(n) + g_z(T^3 - s_w^2 Q) Z_\mu(n) + \frac{g}{\sqrt{2}} \{T_+ W_\mu^+(n) + T_- W_\mu^-(n)\} \right]^2 + \dots
\end{aligned} \tag{2.27}$$

as

$$\begin{aligned}
H_G &= i \int_{p,q} \sum_\mu \bar{\psi}(p) V_{1\mu}(p+q) G_\mu(p-q) \psi(q) \\
&+ \frac{1}{2!} a \int_{p,s,q} \bar{\psi}(p) \sum_{\mu\nu} \delta_{\mu\nu} V_{2\mu}(p+q) G_\mu(s) G_\nu(p-s-q) \psi(q) + \dots
\end{aligned} \tag{2.28}$$

where $V_{1\mu}(p) = \gamma_\mu \cos(p_\mu a/2) - ir T_c \sin(p_\mu a/2)$, $V_{2\mu}(p) = T_c r \cos(p_\mu a/2) - i\gamma_\mu \sin(p_\mu a/2)$ and

$$G_\mu(p) = eQ A_\mu(p) + g_z(T^3 - s_w^2 Q) Z_\mu(p) + \frac{g}{\sqrt{2}} \{T_+ W_\mu^+(p) + T_- W_\mu^-(p)\}. \tag{2.29}$$

Here $Q = T^3 + Y$ and $T_\pm = T^1 \pm iT^2$.

Now we discuss the perturbation theory. The Dirac vacua $|\pm\rangle$ for the free Hamiltonians $\mathcal{H}_\pm(A=0)$ are defined as, $b_+, d_+|+\rangle = 0$ and $b_-, d_-|-\rangle = 0$ and their energy eigenvalues are denoted as $E_\pm(0)$, $\mathcal{H}_\pm(A=0)|\pm\rangle = E_\pm(0)|\pm\rangle$. Then for the Dirac vacua $|A\pm\rangle$, the eigenvalue equations,

$$\mathcal{H}_\pm(A)|A\pm\rangle = E_\pm(A)|A\pm\rangle, \tag{2.30}$$

are solved in the form of the integral equation using the Dirac vacua $|\pm\rangle$ following the standard time independent perturbation theory. The results are,

$$|A\pm\rangle = \alpha_\pm(A) \left[1 - G_\pm(\mathcal{V} - \Delta E_\pm) \right]^{-1} |\pm\rangle, \tag{2.31}$$

where, $\Delta E_\pm = E_\pm(A) - E_\pm(0) = \langle \pm | \mathcal{V} | A\pm \rangle / \langle \pm | A\pm \rangle$ and

$$G_\pm = \sum_n' |n\pm\rangle \frac{1}{E_\pm(0) - E_\pm(n)} \langle n\pm| = \frac{1 - |\pm\rangle\langle\pm|}{E_\pm(0) - H_\pm(0)}, \tag{2.32}$$

In Eq. (2.32), the sum \sum'_n is over all the excited states $|n\pm\rangle$ of $\mathcal{H}_\pm(0)$ and $E_\pm(n)$ denote their energy eigenvalues. The normalization factors $\alpha_\pm(A)$ are determined, up to phases [15], by the normalization conditions of $|A\pm\rangle$ as

$$|\alpha_\pm(A)|^2 = 1 - \langle A \pm | [\mathcal{V} - \Delta E_\pm] G_\pm^2 [\mathcal{V} - \Delta E_\pm] | A \pm \rangle. \quad (2.33)$$

The correlation functions for the ψ , A_ν and Φ fields are obtained in the form of path integral. For example, the two point function for ψ is given by

$$\frac{\int \mathcal{D}\mathcal{A} \langle A + |\Omega(p, q)|A-\rangle e^{-S(A)}}{\int \mathcal{D}\mathcal{A} \langle A + |A-\rangle e^{-S(A)}}, \quad (2.34)$$

and the two point function for the gauge field is given by

$$\frac{\int \mathcal{D}\mathcal{A} \mathcal{D}\Phi A_\mu(s) A_\nu(t) \langle A + |A-\rangle e^{-S(A)}}{\int \mathcal{D}\mathcal{A} \langle A + |A-\rangle e^{-S(A)}}, \quad (2.35)$$

where S is the actions for the gauge fields and Higgs fields, including the effects of the path integral measure, gauge fixing terms and Faddeev-Popov ghost terms. To evaluate Eqs. (2.34) and (2.35), $|A\pm\rangle$ should be expanded in terms of V using Eqs. (2.31) and (2.32), where V is given by the bilinear of the fermion operator. Then the vacuum expectation value is computed from the commutation relations (2.6). Finally, the path integral of over the gauge and scalar fields are performed following the standard perturbation theory. We list the results of such evaluations for $\langle A + |\Omega(p, q)|A-\rangle$ and $\langle A + |A-\rangle$ in appendix A, from which we can compute the fermion and boson two point functions. The propagators of the gauge and Higgs fields are derived in appendix B.

3 Yukawa Interactions

In this section, we study the Yukawa interactions. First we consider the contribution of the electron to the tadpole of the Higgs field

$$\frac{\int \mathcal{D}\mathcal{A} \mathcal{D}\Phi H(s) \langle A + |A-\rangle e^{-S(A)}}{\int \mathcal{D}\mathcal{A} \langle A + |A-\rangle e^{-S(A)}}. \quad (3.1)$$

Here $\langle A + |A-\rangle$ should be expanded to the first order in the Yukawa couplings:

$$\langle A + |A-\rangle = \langle +|VG_+|-\rangle + \langle +|G_-V|-\rangle, \quad (3.2)$$

and V is given by Eq. (2.25) with

$$Y(p-q) = \frac{y}{\sqrt{2}} H(p-q) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (3.3)$$

Using the expressions (A.16) and (A.17) in appendix A,

$$\begin{aligned} \langle +|VG_+|- \rangle + \langle +|G_-V|- \rangle &= - \sum_f \int_p \frac{1}{2\omega_+(p, f)} \text{Tr} Y(0) T_+(p, f) \\ &+ \sum_f \int_p \frac{1}{2\omega_-(p, f)} \text{Tr} Y(0) T_-(p, f), \end{aligned} \quad (3.4)$$

where

$$\text{Tr} Y(0) T_{\pm}(p, f) = \frac{1}{\sqrt{2}} y H(0) \text{Tr} \{ T_{\pm}(p, e)_{LR} + T_{\pm}(p, e)_{RL} \}, \quad (3.5)$$

and the trace in the right-hand side is over the Dirac matrices (The subscripts “ LR ” etc indicate the elements of the matrix propagator T_{\pm} . See Eq. (A.3)). From the explicit expressions for the propagators in the appendix A, we get the tadpole term $H(0)\{t_+ + t_-\}$ with

$$\begin{aligned} t_+ &= \frac{1}{\sqrt{2}} y m_e \int_p \frac{1}{\omega_+} \left[\frac{1}{Z_+} \{ \omega_+ + X_+ + \omega_- + X_- \} - \frac{1}{\omega_+} \right], \\ t_- &= \frac{1}{\sqrt{2}} y m_e \int_p \frac{1}{\omega_-} \left[\frac{1}{Z_-} \{ \omega_- - X_- + \omega_+ - X_+ \} - \frac{1}{\omega_-} \right]. \end{aligned} \quad (3.6)$$

Rescaling the loop momentum as $p \rightarrow \bar{p} = ap$, we find $\omega_{\pm} \rightarrow \tilde{\omega}_{\pm}/a$ and $X_{\pm} \rightarrow \tilde{X}_{\pm}/a$, where

$$\tilde{\omega}_{\pm}(\bar{p}, e) = \sqrt{\tilde{p}^2 + a^2 m_e^2 + \tilde{X}_{\pm}(\bar{p})}, \quad \tilde{X}_{\pm}(\bar{p}) = \pm \lambda + \frac{r}{2} \hat{p}^2, \quad (3.7)$$

with $\tilde{p}_{\mu} = \sin \bar{p}_{\mu}$ and $\hat{p} = 2 \sin(\bar{p}_{\mu}/2)$. So the tadpole term is in fact quadratically divergent; $t_{\pm} = \tilde{t}_{\pm}/a^2$. This term should be canceled by the tadpole counter-term in the action of the Higgs boson (B.6) given in appendix B: $e^{-S} \sim -\delta(\mu^2 + \lambda_{\Phi} v^2) v H(0)$, as

$$-\delta(\mu^2 + \lambda_{\Phi} v^2) v + \frac{1}{a^2} (\tilde{t}_+ + \tilde{t}_-) = 0. \quad (3.8)$$

Because of the global $SU(2)$ symmetry, this quadratic divergence of the tadpole counter-term is related to the mass counter-terms of the Nambu-Goldstone bosons π_{\pm} and π_3 . For example,

$$\frac{1}{a^2} (\tilde{t}_+ + \tilde{t}_-) = v \Sigma_{\pi_3 \pi_3}(0). \quad (3.9)$$

Here the self-energy of π_3 is computed from

$$\frac{\int \mathcal{D}\mathcal{A} \mathcal{D}\Phi \pi_3(s) \pi_3(t) \langle A + |A-\rangle e^{-S(A)}}{\int \mathcal{D}\mathcal{A} \langle A + |A-\rangle e^{-S(A)}}, \quad (3.10)$$

with (up to the one loop level)

$$\begin{aligned} \langle A + |A-\rangle &= \left\{ 1 - \frac{1}{2} \langle + | V G_+^2 V | + \rangle - \frac{1}{2} \langle - | V G_-^2 V | - \rangle \right\} \langle + | - \rangle \\ &+ \langle + | V G_+ G_- V | - \rangle + \langle + | V G_+ V G_+ | - \rangle + \langle + | G_- V G_- V | - \rangle. \end{aligned} \quad (3.11)$$

(In this case, $\langle + | V G_+ | - \rangle$ and $\langle + | G_- V | - \rangle$ in Eq. (A.14) are the one-particle reducible contributions which should be already renormalized, so we did not show them here.)

The interaction V is given by (2.25) with

$$Y(p-q) = \frac{-iy}{\sqrt{2}} \pi_3(p-q) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \quad (3.12)$$

As an example, we compute the term: $\langle + | V G_+ V G_+ | - \rangle$. For the first term of Eq. (A.20),

$$\begin{aligned} \langle + | V G_+ V G_+ | - \rangle &= \frac{1}{4} y^2 \int_{p,q} \left\{ \frac{1}{\omega_+(p,e) + \omega_+(q,e)} \right\}^2 \pi_3(p-q) \pi_3(q-p) \\ &\times \text{Tr} T_+(p,e) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} T_+(q,e) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \end{aligned} \quad (3.13)$$

Inserting this expression into (3.10) and performing the path integral, the bilinears of the π_3 fields should be replaced by its propagator given in appendix B, and we get $(2\pi)^4 \delta_P(s+t) D(s) \Sigma_{\pi_3 \pi_3}(s) D(s)$ with

$$\Sigma_{\pi_3 \pi_3}(s) = \frac{1}{2} y^2 \int_p \left\{ \frac{1}{\omega_+(p,e) + \omega_+(p+s,e)} \right\}^2 \text{Tr} T_+(p,e) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} T_+(p+s,e) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (3.14)$$

Rescaling the loop momentum as $p \rightarrow \bar{p} = ap$, we get the quadratic divergence,

$$\Sigma_{\pi_3 \pi_3}(0) = -\frac{1}{a^2} y^2 \int_p \frac{1}{4\tilde{\omega}_+(\bar{p},e)^2} \text{Tr} \tilde{T}_+(\bar{p},e)_{LL} \tilde{T}_+(\bar{p},e)_{RR}, \quad (3.15)$$

where \tilde{T}_+ is the propagator after the momentum rescaling. Similarly, the second part of Eq. (A.20) yields the following quadratic divergence,

$$\begin{aligned} \Sigma_{\pi_3 \pi_3}(0) &= \frac{1}{a^2} y^2 \int_p \frac{1}{4\tilde{\omega}_+(\bar{p},e)^2} \text{Tr} \left[\tilde{T}_+(\bar{p},e)_{LL} \left\{ \tilde{P}_+(\bar{p},e)_{RR} - \tilde{N}_+(\bar{p},e)_{RR} \right\} \right. \\ &\quad \left. + \tilde{T}_+(\bar{p},e)_{RR} \left\{ \tilde{P}_+(\bar{p},e)_{LL} - \tilde{N}_+(\bar{p},e)_{LL} \right\} \right]. \end{aligned} \quad (3.16)$$

The contributions of the other terms in Eq. (3.11) are computed similarly, and the quadratic divergence of $\Sigma_{\pi_3\pi_3}(0)$ should satisfy the relation (3.9). For simplicity, we confirm this relation in the limit of the Wilson parameter $r \rightarrow 0$. In this limit, $\tilde{\omega}_+ = \tilde{\omega}_-$ and $\tilde{X}_- = -\tilde{X}_+$, and each term in Eq. (3.11) leads to the following contributions to $\Sigma_{\pi_3\pi_3}(0)$,

$$\begin{aligned} \langle +|VG_+VG_+|-\rangle, \quad \langle +|G_-VG_-V|-\rangle &\rightarrow \Sigma_{\pi_3\pi_3}(0) = -\frac{1}{2a^2}y^2 \int_p \left\{ \frac{1}{\tilde{\omega}^2} - \frac{1}{\tilde{p}^2} \right\} \\ -\frac{1}{2}\langle +|VG_+^2V|+\rangle, \quad -\frac{1}{2}\langle -|VG_-^2V|-\rangle &\rightarrow \Sigma_{\pi_3\pi_3}(0) = -\frac{1}{2a^2}y^2 \int_p \frac{1}{\tilde{\omega}^2} \\ \langle +|VG_+G_-V|-\rangle &\rightarrow \Sigma_{\pi_3\pi_3}(0) = \frac{y^2}{a^2} \int_p \frac{1}{\tilde{p}^2} \end{aligned} \quad (3.17)$$

while the tadpole contribution is

$$(t_+ + t_-) = \frac{1}{a^2} 2\sqrt{2} y m_e \int_p \left\{ \frac{1}{\tilde{p}^2} - \frac{1}{\tilde{\omega}^2} \right\}. \quad (3.18)$$

Now it is easy to see that the relation (3.9) is satisfied.

Next we compute the Higgs boson self-energy given by

$$\frac{\int \mathcal{D}\mathcal{A}\mathcal{D}\Phi H(s)H(t)\langle A+|A-\rangle e^{-S(A)}}{\int \mathcal{D}\mathcal{A}\langle A+|A-\rangle e^{-S(A)}}, \quad (3.19)$$

with $\langle A+|A-\rangle$ given by Eq. (3.11). We compute the contribution of the first term of Eq. (A.20). Performing the path integral, it is reduced to the form of $(2\pi)^4 \delta_P(s+t) D(s) \Sigma_{HH}(s) D(s)$ with

$$\Sigma_{HH}(s) = -\frac{1}{2}y^2 \int_p \left\{ \frac{1}{\omega_+(p, e) + \omega_+(p+s, e)} \right\}^2 Tr T_+(p, e) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T_+(p+s, e) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (3.20)$$

Here $\Sigma_{HH}(s)$ contains the quadratic and logarithmic divergences, as well as constant terms. The quadratically divergent part is same to $\Sigma_{\pi_3\pi_3}(0)$. The logarithmic divergence is evaluated by dividing the integration region into two pieces, following Ref. [16]. After the rescaling of the loop momentum $p \rightarrow \bar{p} = ap$, the logarithmic (infrared) divergence occurs in the limit $a \rightarrow 0$ near the center of the Brillouin zone. (The other corners of the Brillouin zone do not leads to the pole of the propagators.) Dividing the integration

region into the vicinity of the center of the Brillouin zone and elsewhere, in the calculation of the former region, the propagators can be expanded in terms of \bar{p} and a , leading to the expression,

$$\begin{aligned}\Sigma_{HH}(s) - \Sigma_{HH}(0) &= \frac{y^2}{2a^2} \int \frac{\bar{p}^2 + a\bar{p}s - a^2m_e^2}{(\bar{p}^2 + a^2m_e^2)\{(\bar{p} + as)^2 + a^2m_e^2\}} - (s=0 \text{ part}) \\ &= \frac{1}{4}y^2\{s^2 + 6m_e^2\} \frac{1}{16\pi^2} \log(a^2\mu^2).\end{aligned}\quad (3.21)$$

The four particle contributions of $\langle +|G_-VG_-V|-\rangle$ (the first term of Eq. (A.21).) leads to the same logarithmic divergence as Eq. (3.21) while the logarithmic divergence of $\langle +|VG_+G_-V|-\rangle$ is the twice of Eq. (3.21). The other terms do not yield any logarithmic divergence. Summing up all the contributions we obtain the final expression,

$$\Sigma_{HH}(s) = \frac{\sigma_\Phi}{a^2} + y^2\{s^2 + 6m_e^2\} \frac{1}{16\pi^2} \log(a^2\mu^2) + (\mu, s, M_H \text{ dependent term}), \quad (3.22)$$

where μ is the renormalization point and σ_Φ/a^2 is the quadratic divergence.

As a final example of the Yukawa interactions, we consider the contribution of the Higgs boson to the electron self-energy:

$$\frac{\int \mathcal{DA} \langle A + |\Omega(p, q)|A-\rangle e^{-S(A)}}{\int \mathcal{DA} \langle A + |A-\rangle e^{-S(A)}}. \quad (3.23)$$

Here the interaction V is given by (2.25) with (3.3). Using Eqs. (A.10) and (A.11), we get

$$\langle +|VG_+VG_+:\Omega(p, q):|-\rangle = -S_-(p, f)\Sigma(p, q)S_+(q, h), \quad (3.24)$$

where

$$\begin{aligned}\Sigma(p, q) &= -(2\pi)^4 \delta_P(p - q) \frac{1}{2} y^2 \int_k D(k - p) \left\{ \frac{1}{\omega_+(p, e) + \omega_+(k, e)} \right\}^2 \begin{pmatrix} T_+(k, e)_{RR} & T_+(k, e)_{RL} \\ T_+(k, e)_{LR} & T_+(k, e)_{LL} \end{pmatrix} \\ &= (2\pi)^4 \delta_P(p - q) \frac{a}{4\lambda} \begin{pmatrix} \Sigma_+(p, e)_{LL} & \Sigma_+(p, e)_{LR} \\ \Sigma_+(p, e)_{RL} & \Sigma_+(p, e)_{RR} \end{pmatrix}.\end{aligned}\quad (3.25)$$

Here each $\Sigma_+(p, e)$ is evaluated as before by rescaling the loop momentum and we obtain

$$\begin{aligned}\Sigma_+(p, e)_{LL(RR)} &= \frac{1}{a} \{\sigma_1 + \gamma_5 \sigma_2\} + P_{R(L)} \frac{1}{4} y^2 i \not{p} \frac{1}{16\pi^2} \log(a^2\mu^2) \\ &\quad + (p, m_e \text{ and } \mu, \text{ dependent term}), \\ \Sigma_+(p, e)_{LR(RL)} &= -P_{R(L)} \frac{1}{2} y^2 m_e \frac{1}{16\pi^2} \log(a^2\mu^2).\end{aligned}\quad (3.26)$$

The other contributions are evaluated in the same way. The four particle contribution (the first term of (A.13)) and $\Sigma(p, q)_\pm$ in Eq. (A.9) give rise to the logarithmic divergences same to Eq. (3.26), while the other terms do not lead any logarithmic divergences. The expression (3.26) shows that the chiral symmetry is preserved after taking into account the Yukawa interactions and the Dirac mass is renormalized multiplicatively.

4 Gauge Interaction

Next we consider the gauge interactions. First we consider the contribution of the electron to the vacuum polarization [1, 10, 11] of the Z boson:

$$\frac{\int \mathcal{D}\mathcal{A}\mathcal{D}\Phi Z_\mu(s)Z_\nu(t)\langle A + |A-\rangle e^{-S(A)}}{\int \mathcal{D}\mathcal{A}\langle A + |A-\rangle e^{-S(A)}}, \quad (4.1)$$

where $\langle A + |A-\rangle$ should be expanded to the second order in the gauge couplings:

$$\begin{aligned} \langle A + |A-\rangle &= \left\{1 - \frac{1}{2}\langle +|VG_+^2V|+\rangle - \frac{1}{2}\langle -|VG_-^2V|-\rangle\right\}\langle +|-\rangle + \langle +|VG_+|-\rangle + \langle +|G_-V|-\rangle \\ &+ \langle +|VG_+G_-V|-\rangle + \langle +|VG_+VG_+|-\rangle + \langle +|G_-VG_-V|-\rangle. \end{aligned} \quad (4.2)$$

First we consider the tadpole type corrections $\langle +|VG_+|-\rangle$ and $\langle +|G_-V|-\rangle$. In this case the interaction V is given by

$$V = \frac{1}{2}ag_Z^2 \int_{p,s,q} \bar{\psi}(p) \sum_{\mu\nu} V_{2\mu}(p+q) \begin{pmatrix} c_n & 0 & 0 \\ 0 & c_L & 0 \\ 0 & 0 & c_R \end{pmatrix} Z_\mu(s)Z_\nu(p-s-q)\psi(q). \quad (4.3)$$

Using the formulae (A.16) and (A.17), the contribution of the electron is given by

$$\begin{aligned} \langle +|VG_+|-\rangle &= \frac{1}{2} \sum_{\mu\nu} \int_s A_\nu(s)A_\mu(s)\Pi_{\mu\nu}(0), \\ \Pi_{\mu\nu}(0) &= -\delta_{\mu\nu}g_Z^2a \int_p \frac{1}{2\omega_+} \text{Tr} V_{2\mu}(2p)\{c_L^2 T_+(p,e)_{LL} + c_R^2 T_+(p,e)_{RR}\}. \end{aligned} \quad (4.4)$$

After rescaling the loop momentum $p \rightarrow \bar{p} = ap$, we find that $\Pi_{\mu\nu}(0)$ is quadratically divergent and is not gauge invariant. This type of contribution should be cancelled when all the terms in Eq. (4.2) are taken into account. At the one-loop level, this is guaranteed by the fact that the real part of the overlap is gauge invariant [1, 10, 11].

Here we demonstrate this cancellation in the limit of the Wilson parameter $r \rightarrow 0$ [17]. In this limit,

$$\langle +|VG_+|- \rangle + \langle +|G_-V|- \rangle \rightarrow \Pi_{\mu\nu}(0) = -\delta_{\mu\nu} \frac{\bar{g}^2}{a^2} \int_{\bar{p}} \frac{\tilde{p}_\mu^2}{\tilde{p}^2} \left\{ \frac{1}{\tilde{\omega}^2} - \frac{1}{\tilde{\omega}^2} \right\}, \quad (4.5)$$

with $\bar{g}^2 = (c_L^2 + c_R^2)g_Z^2$. The other contributions are,

$$\begin{aligned} & -\frac{1}{2} \langle +|VG_+^2V|+ \rangle, \quad -\frac{1}{2} \langle -|VG_-^2V|- \rangle \rightarrow \Pi_{\mu\nu}(0) = -\delta_{\mu\nu} \frac{\bar{g}^2}{4a^2} \int_p c_\mu^2 \left\{ \frac{1}{\tilde{\omega}^2} - \frac{1}{\tilde{\omega}^4} \tilde{p}_\mu^2 \right\}, \\ & \langle +|VG_+VG_+|- \rangle, \quad \langle +|G_-VG_-V|- \rangle \\ & \rightarrow \Pi_{\mu\nu}(0) = \delta_{\mu\nu} \frac{\bar{g}^2}{4a^2} \int_p c_\mu^2 \left\{ \frac{1}{\tilde{p}^2} - \frac{1}{\tilde{\omega}^2} - \frac{2}{\tilde{\omega}^4} \tilde{p}_\mu^2 + \frac{3}{\tilde{\omega}^4} \tilde{p}_\mu^2 - \frac{1}{\tilde{\omega}^2 \tilde{p}^2} \tilde{p}_\mu^2 \right\}, \\ & \langle +|VG_+G_-V|- \rangle \rightarrow \Pi_{\mu\nu}(0) = \delta_{\mu\nu} \frac{\bar{g}^2}{2a^2} \int_p c_\mu^2 \left\{ \frac{1}{\tilde{p}^2} - \frac{2}{\tilde{\omega}^4} \tilde{p}_\mu^2 + \frac{1}{\tilde{\omega}^2 \tilde{p}^2} \tilde{p}_\mu^2 \right\}, \end{aligned} \quad (4.6)$$

where $c_\mu = \cos(\bar{p}_\mu)$. Summing up the contributions (4.6) the terms proportional to $1/(\tilde{\omega}^2 \tilde{p}^2)$ are canceled with each other. For the terms proportional to $c_\mu \tilde{p}_\mu / \tilde{\omega}^4$, using the relation

$$\frac{\partial}{\partial \bar{p}_\mu} \frac{1}{\tilde{\omega}^2} = -\frac{2c_\mu \tilde{p}_\mu}{\tilde{\omega}^4} \quad (4.7)$$

and integrating by part, it becomes the second term of Eq. (4.5) with the opposite sign. In this way, the gauge non-invariant quadratic divergences cancel with each other in the vacuum polarization. The logarithmically divergent part of $\Pi_{\mu\nu}(s)$ is evaluated in a similar way to $\Sigma_{HH}(s)$, and the result is

$$\Pi_{\mu\nu}(s) = g_Z^2 \left[(c_L^2 + c_R^2) \left\{ \frac{2}{3} (s^2 \delta_{\mu\nu} - s_\mu s_\nu) + 2m_e^2 \delta_{\mu\nu} \right\} - 4c_L c_R m_e^2 \delta_{\mu\nu} \right] \frac{1}{16\pi^2} \log(a^2 \mu^2), \quad (4.8)$$

which also satisfies the transverse condition up the spontaneous symmetry breaking factor.

Next we consider the contribution of the Z boson to the electron self-energy. The four particle contribution in $\langle +|VG_+VG_+:\Omega(p, q):|- \rangle$ is given by

$$\begin{aligned} \Sigma(p, q) &= -(2\pi)^4 \delta_P(p - q) \sum_{\mu\nu} e^{-i(p-q)a/2} g_Z^2 \int_k \left\{ \frac{1}{\omega_+(p, e) + \omega_+(k, e)} \right\}^2 V_{1\mu}(p + k) \\ &\times \begin{pmatrix} c_L^2 T_+(p, e_V)_{LL} & c_L c_R T_+(p, e_V)_{LR} \\ c_L c_R T_+(p, e_V)_{RL} & c_R^2 T_+(p, e_V)_{RR} \end{pmatrix} V_{1\nu}(k + p) D_{\mu\nu}(k - p) \end{aligned} \quad (4.9)$$

$$= (2\pi)^4 \delta_P(p - q) \frac{a}{4\lambda} \begin{pmatrix} c_L^2 \Sigma_{LL}(p) & c_L c_R \Sigma_{LR}(p) \\ c_L c_R \Sigma_{RL}(p) & c_R^2 \Sigma_{RR}(p) \end{pmatrix}, \quad (4.10)$$

where

$$\begin{aligned}\Sigma_{LL(RR)}(p) &= \frac{1}{a}\{\sigma_1 + \sigma_2\gamma_5\} + P_{R(L)}i\not{p}\frac{1}{16\pi^2}\log(a^2\mu^2), \\ \Sigma_{LR(RL)}(p) &= P_{R(L)}4m_e\frac{1}{16\pi^2}\log(a^2\mu^2),\end{aligned}\tag{4.11}$$

in the t'Hooft-Feynman gauge ($\alpha_Z = 1$). The other terms in Eq. (A.7) lead to similar contributions, and the logarithmic divergences are obtained only from $\Sigma(p, q)_\pm$ in Eq. (A.9) and the first term of Eq. (A.13). The expression (4.10) with Eq. (4.11) shows that the fermions are correctly renormalized.

5 Discussion

We have developed lattice perturbation theory for the Yukawa and gauge interactions in the overlap formulation taking a realistic example, the Standard Model, and we have analyzed one and two point functions for the fermions, scalars and gauge bosons at the one loop level. We also gave the compact formulae for the self-energies, from which both the divergent and finite parts can be further analyzed at the one loop level.

Even though we established the lattice perturbation theory for realistic theories and demonstrated the analysis up to the one loop level, there remain issues still to be clarified and which did not appear in our analysis. The unclear point for a regularization of a general chiral gauge theory is the role of the imaginary part of $\langle A + |A-\rangle$ (or equivalently, the phase convention of the vacuum states $|A\pm\rangle$), which can be a functional of the gauge fields (such an effect did not appear at the one loop level). For the anomaly free case, such effects are suppressed by a positive power of the lattice spacing in the expression for $\langle A + |A-\rangle$. However, as is shown in the calculation of the vacuum polarization, the interactions suppressed by a positive power of the lattice spacing play important roles in keeping the gauge invariance at loop levels, and the imaginary part of $\langle A + |A-\rangle$, even if suppressed by a positive power of the lattice spacing, may affect the gauge invariance at higher orders.

For Higgs models and vector-like theories there will be no such subtle points and our analysis suggests that this formulation is feasible for regularizing Higgs models, as well as vector like gauge theory [18], with the advantage of respecting the chiral symmetry.

Acknowledgment

I would like to thank Y. Kikukawa and S. Randjbar-Daemi for discussions, and G. Gyku and M. O’Loughlin for comments on the manuscript.

Appendix A.

In this appendix, we present the general formulae to compute the self-energies of the fermions and bosons (both scalar and vector bosons) in the overlap formulation at the one loop level. First we list the propagators appearing in the expression:

$$\begin{aligned}
S_+(p, f) &= \frac{1}{\cos \beta(p, f)} \sum_{\sigma_f} u_+(p, f, \sigma_f) \bar{u}_-(p, f, \sigma_f), \\
S_-(p, f) &= \frac{1}{\cos \beta(p, f)} \sum_{\sigma_f} v_-(p, f, \sigma_f) \bar{v}_+(p, f, \sigma_f), \\
P_\pm(p, f) &= \sum_{\sigma_f} u_\pm(p, f, \sigma_f) \bar{u}_\pm(p, f, \sigma_f), \quad N_\pm(p, f) = \sum_{\sigma_f} v_\pm(p, f, \sigma_f) \bar{v}_\pm(p, f, \sigma_f).
\end{aligned} \tag{A.1}$$

In the above, S_\pm develop poles near the center of the Brillouin zone. The projection operator $P_\pm(N_\pm) = \{\mathbf{1}_f \omega_\pm(p, f) + (-)H_\pm(p, f)\} \gamma_5 / 2\omega_\pm(p, f)$ do not have any poles. (Here $\mathbf{1}_f = \text{diag}(0, 1, 1)$ for the electron, for example, and $H_\pm(p, f)$ is the block diagonalized part of the one particle Hamiltonian (2.12) acting on the flavor f .) We find it convenient to introduce other two propagators:

$$\begin{aligned}
T_+(p, f) &= \frac{\sin \beta(p, f)}{\cos \beta(p, f)} \sum_{\sigma_f} u_+(p, f, \sigma_f) \bar{v}_+(p, f, \sigma_f) \\
&= -S_+(p, f) + P_+(p, f) = S_-(p, f) - N_-(p, f), \\
T_-(p, f) &= \frac{\sin \beta(p, f)}{\cos \beta(p, f)} \sum_{\sigma_f} v_-(p, f, \sigma_f) \bar{u}_-(p, f, \sigma_f) \\
&= -S_-(p, f) + P_-(p, f) = S_+(p, f) - N_+(p, f).
\end{aligned} \tag{A.2}$$

Explicit expressions of the S_\pm , P_\pm and N_\pm for the Dirac type propagators are,

$$S_\pm(p, f_V) = \frac{1}{Z_\pm} \begin{pmatrix} K_\pm(p, f_V)_{LL} & M_\pm(p, f_V)_{LR} \\ M_\pm(p, f_V)_{RL} & K_\pm(p, f_V)_{LL} \end{pmatrix}, \tag{A.3}$$

where

$$\begin{aligned}
Z_{\pm} &= (\omega_+ \pm X_+)(\omega_- \pm X_-) + \tilde{p}^2 + m_f^2, \\
K_+(p, f_V)_{LL(RR)} &= P_{L(R)}(\omega_+ + X_+) \left\{ -i\tilde{\not{p}} + T_{f_{L(R)}}(\omega_- + X_-) \right\} \\
&\quad + P_{R(L)}(\omega_- + X_-) \left\{ -i\tilde{\not{p}} - T_{f_{L(R)}}(\omega_- - X_-) \right\}, \\
M_+(p, f_V)_{LR(RL)} &= P_{L(R)}m_f(\omega_+ + X_+) + P_{R(L)}m_f(\omega_- + X_-), \tag{A.4}
\end{aligned}$$

$$\begin{aligned}
P_{\pm}(p, f) &= \frac{1}{2\omega_{\pm}(p, f)} \begin{pmatrix} \omega_{\pm}(p, f)\gamma_5 - i\tilde{\not{p}} + T_{f_L}X_{\pm} & m_f \\ m_f & \omega_{\pm}(p, f)\gamma_5 - i\tilde{\not{p}} + T_{f_R}X_{\pm} \end{pmatrix}, \\
N_{\pm}(p, f) &= \frac{1}{2\omega_{\pm}(p, f)} \begin{pmatrix} \omega_{\pm}(p, f)\gamma_5 + i\tilde{\not{p}} - T_{f_L}X_{\pm} & -m_f \\ -m_f & \omega_{\pm}(p, f)\gamma_5 + i\tilde{\not{p}} - T_{f_R}X_{\pm} \end{pmatrix}. \tag{A.5}
\end{aligned}$$

Now we show the expressions of $\langle A+|:\Omega(p, q):|A-\rangle$ and $\langle A+|A-\rangle$ which are necessary to compute the fermion and boson self-energies. To compute these factors, we only consider the one-particle irreducible contributions, and we use the interaction vertex $\Gamma(p, q)$ (which is of the matrix form) defined by,

$$V = \int_{p, q} \bar{\psi}(p) \Gamma(p, q) \psi(q). \tag{A.6}$$

For the calculation up to the one loop level, $\langle A+|:\Omega(p, q):|A-\rangle$ is expanded up to the second order in V :

$$\begin{aligned}
\langle A+|:\Omega(p, q):|A-\rangle &= \langle +|VG_+:\Omega(p, q):|-\rangle + \langle +|:\Omega(p, q):G_-V|-\rangle + \langle +|VG_+:\Omega(p, q):G_-V|-\rangle \\
&\quad + \langle +|VG_+VG_+:\Omega(p, q):|-\rangle + \langle +|:\Omega(p, q):G_-VG_-V|-\rangle, \tag{A.7}
\end{aligned}$$

where the first two terms are evaluated as,

$$\begin{aligned}
\langle +|VG_+:\Omega(p, q):|-\rangle &= S_-(p, f) \left\{ \frac{1}{\omega_+(p, f) + \omega_+(q, h)} \right\} \Gamma(p, q) S_+(q, h), \\
\langle +|:\Omega(p, q):G_-V|-\rangle &= S_+(p, f) \left\{ \frac{1}{\omega_-(p, f) + \omega_-(q, h)} \right\} \Gamma(p, q) S_-(q, h). \tag{A.8}
\end{aligned}$$

The third term is,

$$\langle +|VG_+:\Omega(p, q):G_-V|-\rangle = S_+(p, f)\Sigma(p, q)_+S_+(q, h) + S_-(p, f)\Sigma(p, q)_-S_-(q, h),$$

$$\begin{aligned}
\Sigma(p, q)_+ &= \sum_g \int_k \left\{ \frac{1}{\omega_-(p, f) + \omega_-(k, g)} \right\} \left\{ \frac{1}{\omega_+(k, g) + \omega_+(q, h)} \right\} \Gamma(p, k) \{-S_-(k, g)\} \Gamma(k, q), \\
\Sigma(p, q)_- &= \sum_g \int_k \left\{ \frac{1}{\omega_+(p, f) + \omega_+(k, g)} \right\} \left\{ \frac{1}{\omega_-(k, g) + \omega_-(q, h)} \right\} \Gamma(p, k) S_+(k, g) \Gamma(k, q).
\end{aligned} \tag{A.9}$$

To evaluate these terms, only the two particle states contribute in the sum of the excited states of G_\pm . To evaluate the last two terms in Eq. (A.7), both two and four particle states have to be taken into account in G_\pm . The results are,

$$\langle + | V G_+ V G_+ : \Omega(p, q) : | - \rangle = -S_-(p, f) \Sigma(p, q) S_+(q, h), \tag{A.10}$$

$$\begin{aligned}
\Sigma(p, q) &= - \sum_g \int_k \left\{ \frac{1}{\omega_+(p, f) + \omega_+(k, g)} \right\} \left\{ \frac{1}{\omega_+(k, g) + \omega_+(q, h)} \right\} \Gamma(p, k) T_+(k, g) \Gamma(k, q) \\
&\quad - \left\{ \frac{1}{\omega_+(p, f) + \omega_+(q, h)} \right\} \sum_g \int_k \Gamma(p, k) \left\{ - \frac{P_+(k, g)}{\omega_+(p, f) + \omega_+(k, g)} + \frac{N_+(k, g)}{\omega_+(k, g) + \omega_+(q, h)} \right\} \Gamma(k, q).
\end{aligned} \tag{A.11}$$

$$\langle + | : \Omega(p, q) : G_- V G_- V | - \rangle = S_+(p, f) \Sigma(p, q) \{-S_-(q, h)\}, \tag{A.12}$$

$$\begin{aligned}
\Sigma(p, q) &= \sum_g \int_k \left\{ \frac{1}{\omega_-(p, f) + \omega_-(k, g)} \right\} \left\{ \frac{1}{\omega_-(k, g) + \omega_-(q, h)} \right\} \Gamma(p, k) T_-(k, g) \Gamma(k, q) \\
&\quad - \left\{ \frac{1}{\omega_-(p, f) + \omega_-(q, h)} \right\} \sum_g \int_k \Gamma(p, k) \left\{ - \frac{P_-(k, g)}{\omega_-(p, f) + \omega_-(k, g)} + \frac{N_-(k, g)}{\omega_-(k, g) + \omega_-(q, h)} \right\} \Gamma(k, q).
\end{aligned} \tag{A.13}$$

In the above expressions the first terms of $\Sigma(p, q)$ are the contributions of the four particle states, while the second parts are those of the two particle states.

To compute the boson self-energies in the one-loop, $\langle A + | A - \rangle$ should be expanded up to the second order in V :

$$\begin{aligned}
\langle A + | A - \rangle &= \left\{ 1 - \frac{1}{2} \langle + | V G_+^2 V | + \rangle - \frac{1}{2} \langle - | V G_-^2 V | - \rangle \right\} \langle + | - \rangle + \langle + | V G_+ | - \rangle + \langle + | G_- V | - \rangle \\
&\quad + \langle + | V G_+ G_- V | - \rangle + \langle + | V G_+ V G_+ | - \rangle + \langle + | G_- V G_- V | - \rangle,
\end{aligned} \tag{A.14}$$

where the first term proportional to $\langle + | - \rangle$ comes from the normalization factors $\alpha_\pm(A)$.

Each term gives rise to the following expression:

$$\langle + | V G_+ | - \rangle = \sum_g \int_k (-1) \frac{1}{2\omega_+(k, g)} \text{Tr} \Gamma(k, k) T_+(k, g), \tag{A.15}$$

$$\langle + | G_- V | - \rangle = \sum_g \int_k \frac{1}{2\omega_-(k, g)} \text{Tr} \Gamma(k, k) T_-(k, g), \tag{A.16}$$

$$\langle +|VG_+^2V|-\rangle = \sum_{f,h} \int_{p,q} \left\{ \frac{1}{\omega_+(p,f) + \omega_+(q,h)} \right\}^2 Tr P_+(p,f) \Gamma(p,q) N_+(q,h) \Gamma(q,p), \quad (\text{A.17})$$

$$\langle -|VG_-^2V|-\rangle = \sum_{f,h} \int_{p,q} \left\{ \frac{1}{\omega_-(p,f) + \omega_-(q,h)} \right\}^2 Tr P_-(p,f) \Gamma(p,q) N_-(q,h) \Gamma(q,p), \quad (\text{A.18})$$

$$\begin{aligned} \langle +|VG_+G_-V|-\rangle &= \sum_{f,h} \int_{p,q} \left\{ \frac{1}{\omega_+(p,f) + \omega_+(q,h)} \right\} \left\{ \frac{1}{\omega_-(p,f) + \omega_-(q,h)} \right\} \\ &\times Tr S_+(p,f) \Gamma(p,q) S_-(p,f) \Gamma(q,p), \end{aligned} \quad (\text{A.19})$$

$$\begin{aligned} \langle +|VG_+VG_+|-\rangle &= -\frac{1}{2} \sum_{f,h} \int_{p,q} \left\{ \frac{1}{\omega_+(p,f) + \omega_+(q,h)} \right\}^2 Tr T_+(p,f) \Gamma(p,q) T_+(q,h) \Gamma(q,p) \\ &+ \sum_{f,h} \int_{p,q} \frac{1}{2\omega_+(p,f)} \left\{ \frac{1}{\omega_+(p,f) + \omega_+(q,h)} \right\} Tr T_+(p,f) \Gamma(p,q) \{P_+(q,h) - N_+(q,h)\} \Gamma(q,p), \end{aligned} \quad (\text{A.20})$$

$$\begin{aligned} \langle +|G_-VG_-V|-\rangle &= -\frac{1}{2} \sum_{f,h} \int_{p,q} \left\{ \frac{1}{\omega_-(p,f) + \omega_-(q,h)} \right\}^2 Tr T_-(p,f) \Gamma(p,q) T_-(q,h) \Gamma(q,p) \\ &+ \sum_{f,h} \int_{p,q} \frac{1}{2\omega_-(p,f)} \left\{ \frac{1}{\omega_-(p,f) + \omega_-(q,h)} \right\} Tr T_-(p,f) \Gamma(p,q) \{N_-(q,h) - P_-(q,h)\} \Gamma(q,p). \end{aligned} \quad (\text{A.21})$$

In the above expressions, the trace is over the flavor space and the Dirac matrices. In Eqs. (A.20) and (A.21), the first terms are the contribution of the four particle states in G_\pm , while the second terms are those of the two particle states. The contributions of the other terms are only from the two particle states.

Appendix. B. Gauge and Higgs sector

In this appendix we briefly give the lattice formulation of the $SU(2) \times U(1)$ Higgs doublet. Further discussions are found e.g., in Refs. [19, 20, 21]. The action of the Higgs doublets

$$\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix} = \begin{pmatrix} i\pi_+(x) \\ \{v + H(x) - i\pi_3(x)\}/\sqrt{2} \end{pmatrix} \quad (\text{B.1})$$

is given by

$$S_E = \int d^4x [\Phi^\dagger(x)(-\square + \mu^2)\Phi(x) + \lambda_\Phi \{\Phi^\dagger(x)\Phi(x)\}^2]. \quad (\text{B.2})$$

The gauge invariant lattice action is obtained by discretising it and inserting the link variable:

$$S_E = a^4 \sum_n \left[-\frac{1}{a^2} \sum_\mu 2\text{Re}\{\Phi^\dagger(n)W_{n,n+\hat{\mu}}\Phi(n+\hat{\mu})\} + \frac{1}{a^2} 2d\Phi^\dagger(n)\Phi(n) + \lambda_\Phi \{\Phi^\dagger(n)\Phi(n)\}^2 \right]. \quad (\text{B.3})$$

The lattice action of the gauge fields is given by

$$S_G = \frac{2N}{g^2} \sum_P \left\{ 1 - \frac{1}{2N} \text{Tr}(U_P + U_P^\dagger) \right\} + \frac{1}{g'^2} \sum_P \left\{ 1 - \frac{1}{2} \text{Tr}(V_P + V_P^\dagger) \right\}, \quad (\text{B.4})$$

where U_P and V_P stand for the products of the link variables around a plaquette P , the sum is over the plaquettes and $N = 2$ for $SU(2)$. Expanding the link variables in terms of the gauge couplings in the action (B.3), we find the mass eigenstates of the neutral gauge bosons as $Z_\mu(n) = \cos \theta_W W_\mu^3(n) - \sin \theta_W B_\mu(n)$ and $A_\mu(n) = \sin \theta_W W_\mu^3(n) + \cos \theta_W B_\mu(n)$ with $\tan \theta_W = g'/g$. For the weak coupling expansion, it is convenient to take the covariant gauge with the gauge fixing term,

$$\begin{aligned} S_{GF} &= \frac{1}{\alpha_W} a^4 \sum_n \left\{ \sum_\mu \partial_\mu^L W_\mu^+(n) + \alpha_W M_W \pi_+(n) \right\}^\dagger \left\{ \sum_\mu \partial_\mu^L W_\mu^+(n) + \alpha_W M_W \pi_+(n) \right\} \\ &+ \frac{1}{2\alpha_Z} a^4 \sum_n \left\{ \sum_\mu \partial_\mu^L Z_\mu(n) + \alpha_Z M_Z \pi_3(n) \right\}^2 + \frac{1}{2\alpha_A} a^4 \sum_n \left\{ \sum_\mu \partial_\mu^L A_\mu(n) \right\}^2, \end{aligned} \quad (\text{B.5})$$

where ∂_μ^L and ∂_μ^R are the left and right lattice derivatives. Taking into account the lattice action of the Higgs fields, gauge fields and the gauge fixing terms, we obtain following free parts,

$$\begin{aligned} S &= a^4 \sum_n \left\{ (\mu^2 + \lambda_\Phi v^2) \left\{ vH(n) + \frac{1}{2}H^2(n) + \frac{1}{2}\pi_3^2(n) + \pi_+(n)\pi_-(n) \right\} \right. \\ &+ \frac{1}{2}H(n) \left(-\sum_\rho \partial_\rho^L \partial_\rho^R + M_H^2 \right) H(n) + \frac{1}{2}\pi_3(n) \left(-\sum_\rho \partial_\rho^L \partial_\rho^R + \alpha_Z M_Z^2 \right) \pi_3(n) \\ &+ \pi_-(n) \left(-\sum_\rho \partial_\rho^L \partial_\rho^R + \alpha_W M_W^2 \right) \pi_+(n) + \frac{1}{2}Z_\mu(n) G_{\mu\nu}(\alpha_Z, M_Z) Z_\mu(n) \\ &\left. + \frac{1}{2}A_\mu(n) G_{\mu\nu}(\alpha_A, 0) A_\mu(n) + W_\mu^-(n) G_{\mu\nu}(\alpha_W, M_W) W_\mu^+(n) \right\}, \end{aligned} \quad (\text{B.6})$$

where

$$G_{\mu\nu}(\alpha, M) = \delta_{\mu\nu}(-\sum_{\rho} \partial_{\rho}^L \partial_{\rho}^R + M^2) + (1 - \frac{1}{\alpha}) \partial_{\mu}^R \partial_{\nu}^L. \quad (\text{B.7})$$

The path integral measure and the Faddeev-Popov ghost terms are not necessary in our analysis and do not present here. The propagators are computed from the action (B.6). Performing the Fourier transformations (see, Eq. (2.10)), we obtain the propagator for scalars as

$$\langle \phi(p) \phi^{\dagger}(q) \rangle = (2\pi)^4 \delta_P(p+q) D(p, M), \quad D(p, M) = \frac{1}{\hat{p}^2 + M^2}. \quad (\text{B.8})$$

For the gauge fields,

$$\begin{aligned} \langle A_{\mu}(p) A_{\nu}(q) \rangle &= (2\pi)^4 \delta_P(p+q) D_{\mu\nu}(p, 0) e^{-i(p+q)_{\nu} a/2}, \\ \langle Z_{\mu}(p) Z_{\nu}(q) \rangle &= (2\pi)^4 \delta_P(p+q) D_{\mu\nu}(p, M_Z) e^{-i(p+q)_{\nu} a/2}, \\ \langle W_{\mu}^{+}(p) W_{\nu}^{-}(q) \rangle &= (2\pi)^4 \delta_P(p+q) D_{\mu\nu}(p, M_W) e^{-i(p+q)_{\nu} a/2}, \\ D_{\mu\nu}(p, M) &= D(p, M) \left\{ \delta_{\mu\nu} - (1 - \alpha) \frac{\hat{p}_{\mu} \hat{p}_{\nu}}{\hat{p}^2 + \alpha M^2} \right\}. \end{aligned} \quad (\text{B.9})$$

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